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TRANSIENT PERFORMANCE OF PERIODIC FLOW REGENERATORS

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Abstract—The instantaneous response by a thermal regenerator to step changes in operation is prevented by the thermal inertia of the system. The transient response of periodic flow regenerators to simultaneous step changes in inlet gas temperature and gas flow rate are examined. It is shown how the transient response to a single step change in one of these two input parameters can be superimposed upon that of the other in order to obtain the response of simultaneous step changes.

NOMENCLATURE

A,	regenerator heating surface area [m^2];
C,	specific heat of storing matrix [J/kg, K];
$\bar{h}$,	bulk heat-transfer coefficient [W/m^2, K];
M,	mass of heat-transfer matrix [kg];
m,	mass of gas resident in regenerator [kg];
P,	length of operating period [s];
S,	specific heat of gas [J/kg, K];
T,	temperature of heat storing matrix [K];
t,	temperature of gas [K];
W,	flow rate of gas [kg/s].

Greek symbols

β,	degree of unbalance $\Lambda'\Pi'/\Lambda''\Pi''$;
ξ,	dimensionless length;
$\varepsilon_{g1}, \varepsilon_{g2}$,	dimensionless measures of transient response produced by step change in inlet gas temperature;
η,	dimensionless time;
Λ,	reduced length $\bar{h}A/WS$ [dimensionless];
Π,	reduced period $\bar{h}A(P-m/W)/MC$ [dimensionless].

Subscripts

IN,	inlet;
x	mean exit.

Superscripts

$'$,	refers to hot period;
$''$,	refers to cold period;
$*$,	refers to new equilibrium;
0,	refers to old equilibrium;
$^-$,	refers to effect of simultaneous step changes obtained by superposition.

INTRODUCTION

THE STUDY of London *et al.* [1] of the transient response of the exit gas temperatures from a thermal regenerator arising from step input changes in inlet gas temperature has been extended recently by Willmott and Burns [2] to the response to step changes in gas flow rate. Whereas London *et al.* restricted their considerations to balanced rotary regenerators for cases where the angular velocity of the rotor is great relative to the gas flow rate, Willmott and Burns examined fixed bed and rotary regenerators over a wide range of operating parameters for both the balanced and unbalanced cases. However, step changes in inlet gas temperature were treated separately from those in gas flow rate and the purpose of this paper is to examine the transient response of a regenerator to simultaneous step changes in these two operating parameters.

Cowper stoves are required to meet a time varying thermal load from the ironmaking furnace. The control strategies that must be developed if these varying demands are to be accommodated with minimal fuel consumption may well involve simultaneous changes in the flow rate and inlet temperature of the gas passing through this type of regenerator. It seems likely that some form of regenerative apparatus will be embodied in solar energy collection systems when it is required on the one hand to accommodate during the warmer seasons of the year a minimum thermal load and on the other to smooth out the effect of inevitable short term variations in the weather. An understanding of the thermal inertia of regenerators including the response to simultaneous step changes in the inlet temperature and flow rate of the gas is fundamental in these areas of use of thermal regenerators.

Willmott and Burns showed in their computer simulation studies that for single step changes in either gas inlet temperature or gas flow rate considered separately, the transient response can be characterised by the harmonic mean of the reduced lengths Λ_H and a parameter β, which measures the degree of regenerator unbalance. The total time to re-establish equilibrium can be regarded as independent of period length and can be made up of many short cycles or a few long cycles. The observation of London *et al.* [1] for balanced regenerators was confirmed, namely that the thermal inertia increases with reduced length. Willmott and Burns showed that for a particular harmonic mean reduced length Λ_H, any unbalance in the system reduces the time required for cyclic equilibrium to be re-established.

MATHEMATICAL REPRESENTATION OF STEP CHANGES IN OPERATION

The temperature behaviour of a thermal regenerator is represented by the differential equations

$$\frac{\partial t}{\partial \xi} = T - t \tag{1}$$

$$\frac{\partial T}{\partial \eta} = t - T \tag{2}$$

discussed by Hausen [3] and described by Willmott and Burns [2]. The assumptions embodied in this model are discussed by Willmott and Thomas [4]. The following boundary condition appertains.

> The inlet gas temperatures and gas flow rates of both the hot and cold periods remain constant during any cycle. Step changes in operation are made either simultaneously at the beginning of the period under consideration or at the beginning of opposite periods of the same cycle. All operating conditions then remain unchanged until cyclic equilibrium is re-established.

Hausen proposed the dimensionless parameters "reduced length" Λ and "reduced period" Π to characterise each period of operation, where

$$\Lambda = \frac{\bar{h}A}{WS} \tag{3}$$

$$\Pi = \frac{\bar{h}A}{MC}\left(P - \frac{m}{W}\right). \tag{4}$$

It is common to use the superscripts $'$ and $''$ to distinguish hot and cold periods. By considering the case where $\bar{h}$ is approximately linearly proportional to W, the response of the regenerator following a step change in hot period gas flow rate can be investigated by considering step changes in Π' alone. This has been shown [5] to have a non-significant effect upon the transient behaviour of a regenerator with reduced length $\Lambda > 5$. Similarly step changes in W'' are approximated by step changes in Π''.

The linear nature of equations (1) and (2) enables considerable overall simplification of any consideration of the transient response of the system to step changes in operation. Iliffe [6] demonstrated that cyclic equilibrium temperature performance was independent of the temperature scale prescribed by the hot and cold period inlet gas temperatures. In exactly the same way the transient behaviour of a regenerator can be placed on a dimensionless temperature scale independent of the initial and final inlet gas temperatures. Willmott and Burns [2] illustrated the transient responses of the regenerator by plotting, cycle by cycle, the time mean exit gas temperatures for both the hot and cold periods. Furthermore, although there is a clear difference between the hot and cold periods in the physical system, this distinction is irrelevant in terms of the linear mathematical representation. As a consequence the results presented here are more general than they appear since the hot and cold periods can be interchanged without any changes in the transient responses computed using this model.

More important, the effects of simultaneous or multiple step changes in the inlet gas temperatures can be superimposed upon one another by combining linearly the effect of each individual step change. This follows from the linearity of equations (1) and (2) and the fact that the initial (when a step change occurs) temperature conditions and the final (when cyclic equilibrium is restored) conditions for the separate step change are superimposable upon one another. Moreover this superposition approach can be extended to include the effect of a single non-linear step change in gas flow rate, provided the step changes in inlet gas temperature takes place after the one in flow rate. This is necessary so that the quantities which enter non-linearly into the equations (e.g. gas flow rate) should be constant throughout the response period.

A possible way of looking at this superposition is to consider the response resulting from a step change in inlet gas temperature to be a "transient rescaling" of the exit gas temperatures from the old $t'_{IN} - t''_{IN}$ scale to the new $t'^{*}_{IN} - t''_{IN}$ one. For example if t'_x and t''_x represent the transient response of a regenerator following a step change in gas flow rate (typical responses are illustrated in Fig. 1). Then the effect $\bar{t}'_x$, $\bar{t}''_x$ of a simultaneous step change in inlet gas temperature will be given by

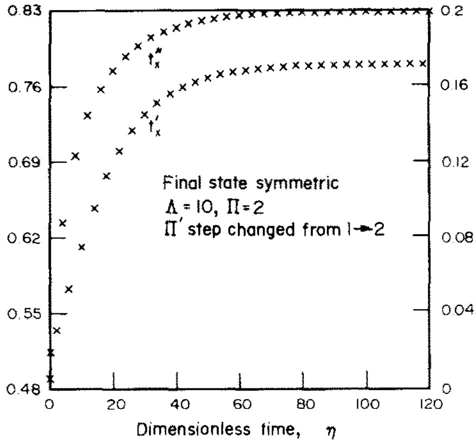

FIG. 1. Responses t'_x, t''_x of a regenerator to a step change in hot gas flow rate.

$$\bar{t}'_x = t'_x + \varepsilon_{g1}(t'^{*}_x - t'^{0}_x) \tag{5}$$

$$\bar{t}''_x = t''_x + \varepsilon_{g2}(t''^{*}_x - t''^{0}_x) \tag{6}$$

where ε_{g1} and ε_{g2} are the transient responses of the regenerator following a step change in inlet gas temperature alone, placed on a (0–1) scale and t^{*}_x, t^{0}_x represent the cyclic equilibrium exit gas temperatures for the new and old temperature scales respectively as measured with the new gas flow rate.

The advantage of having the inlet gas temperatures' responses on a (0–1) scale is due to the fact that the magnitude of the step change has only a linear effect

Table 1.

	Case I Before	Case I After	Case II Before	Case II After	Case III Before	Case III After
Reduced length (hot period) Λ'	10	10	12	12	12	12
Reduced length (cold period) Λ''	10	10	10	10	10	10
Reduced period (hot period) Π'	1	2	3	2	2	2
Reduced period (cold period) Π''	2	2	1	1	1	2
Hot inlet gas temperature t'_{IN}	1	1.5	1	1.5	1	1.5
Cold inlet gas temperature t''_{IN}	0	0	0	0	0	0
	Final state is *symmetric*		Step change in opposite directions *unbalanced*		Step changes is W'' and t'_{IN} *unbalanced*	

upon the transient response. Therefore only one set of results are required to consider a large number of possible step changes appertaining to any particular gas flow rate.

The superimposition method described here has been used to investigate a wide range of simultaneous or multiple step changes in inlet gas temperature together with single step changes in gas flow rate. Three test examples are given in Table 1 and are illustrated in Figs. 2–4. The superimposed results are shown as points joined by straight lines, the actual responses obtained by direct simulation are also indicated, as starred points.

It is important to note that in both Case II and Case III the responses are no longer bounded by the initial and final equilibrium positions.

DISCUSSION

The thermal behaviour of regenerative heat exchangers can be computed directly for cyclic equilibrium by closed methods such as those of Iliffe [6] and

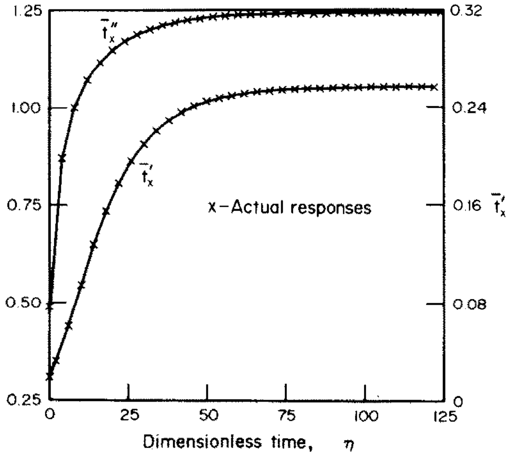

FIG. 2. Comparison of predicted and actual responses—case I.

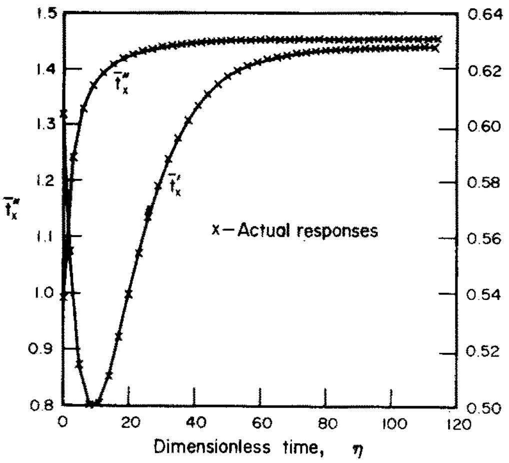

FIG. 3. Comparison of predicted and actual responses—case II.

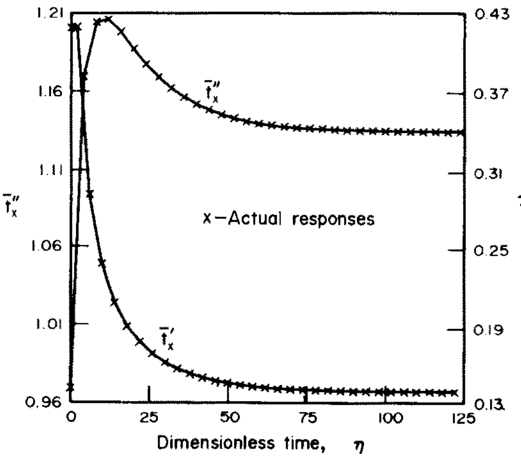

FIG. 4. Comparison of predicted and actual responses—case III.

Nahavandi and Weinstein [7]. Solutions obtained by such methods for the initial and final states of a regenerator undergoing a change in operation provide bounds for the variations in exit gas temperature for single step changes in inlet gas temperature or gas flow rate. These same bounds apply where simultaneous increases (or simultaneous decreases) are made to the flow rate and inlet gas temperature *in the same period of operation* in the cycle.

However, when an increase in inlet temperature is accompanied by a decrease in flow rate (or vice-versa) and/or when the change in one operating parameter is accompanied by a change in another parameter *in the opposite period*, these bounds may no longer apply. Illustrative responses $\bar{t}'_x$ and $\bar{t}''_x$ for cases II and III are displayed in Figs. 3 and 4.

For the decrease in hot period flow rate and the simultaneous increase in inlet temperature of case II, there is a sharp and possibly unexpected decrease in $\bar{t}'_x$ before a steady move towards equilibrium begins. Meanwhile, $\bar{t}''_x$ experiences no such parallel effects and progression of $\bar{t}''_x$ towards its equilibrium value is rapid. A similar but less severe effect is noted for case III where $\bar{t}''_x$ initially "overshoots" its equilibrium value after the simultaneous increase of cold period flow rate and hot inlet gas temperature have been imposed.

Willmott and Burns [2] observed that the response of the hot period exit gas temperature to a step change in either inlet gas temperature or gas flow rate is more rapid than that of the cold period exit gas temperature if the step change occurs in the cold period and vice-versa. In other words, most rapid response is obtained in the "opposite" period of the cycle to that in which the single step change is made.

When a step change in, say, hot period inlet gas temperature is accompanied by an increase in cold period gas flow rate, it can be anticipated from the superimposition possible of one response upon another described here, that the effect of the change in hot inlet gas temperature becomes manifest in the cold period exit gas temperature far more rapidly than does the effect of the other step change. However the responses to changes in gas flow rate are functions of the size of step change. It follows that if the increase in cold period gas flow rate is sufficiently large, as its effect becomes manifest, it will "pull back" the exit gas temperature in the cold period. For small step changes, this pulling back becomes less marked and in Fig. 5, the combined response $\bar{t}''_x$ is displayed for decreasing size of step change in gas flow rate. The curves $\bar{t}''_x$ approach that of t''_x as the magnitude of flow rate change becomes smaller, where t''_x is the response to a step change in hot side inlet gas temperature alone.

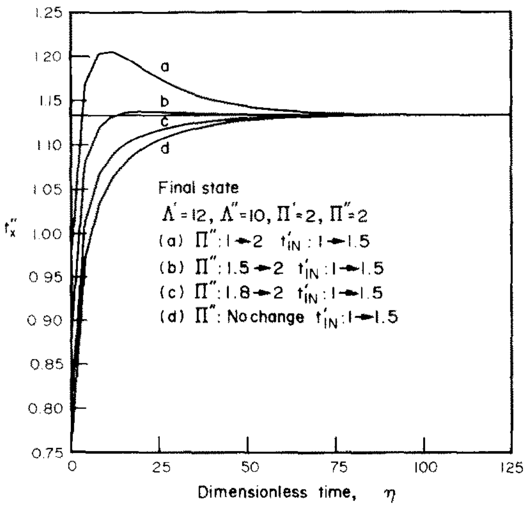

FIG. 5. Dependence of response $\bar{t}''_x$ upon magnitude of the step change in Π''.

By way of further illustration, a regenerator of large Cowper stove dimensions is considered and specified to be case IV.

Table 2. Case IV

Reduced length	Before change	After change
Λ'	15	15
Λ''	15	15
Reduced period		
Π'	1	1
Π''	1.2	0.9
Inlet gas temperature		
t'_{IN}	1400	1217
t''_{IN}	150	150

A Cowper stove is fired in the hot period by "blast furnace gas" which is a surplus product from the ironmaking process. However the calorific value of this gas has decreased over recent years as blast furnace operation has become more efficient. As a consequence, the maximum inlet gas temperature to the stove in the hot period is limited by the highest combustion temperature of the blast furnace gas which can be achieved and it has become necessary to enrich blast furnace gas with expensive fuels such as oil and natural gas. Case IV corresponds very approximately to the circumstances where a decrease in blast flow rate (cold period) is accommodated by a decrease in the proportion of fuels enriching the blast furnace gas thus leading to a decrease in inlet gas temperature to the stove in the hot period. The response of the stove is an immediate decrease in the cold period mean exit temperature from 1134 to 1079°C followed ty a gradual return to the original value of 1134°C. This is illustrated in Fig. 6; note that the hot period mean exit temperature steadily increases towards its final equilibrium state.

CONCLUSION

The transient effects illustrated by cases II–IV point towards some of the difficulties of managing regenerative heat storage systems and of operating industrial regenerators with minimum fuel con-

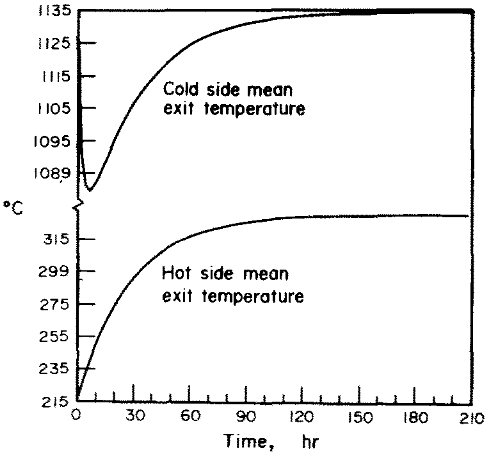

FIG. 6. Response of a regenerator of Cowper stove dimensions—case IV.

sumption under variable thermal load conditions. However the means described here whereby the effect of a step change in inlet gas temperature can be superimposed upon the effect of a step change in gas flow rate provides one basis for the overcoming of these difficulties.

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PERFORMANCE TRANSISTOIRE DE REGENERATEURS A ECOULEMENT PERIODIQUE

Résumé—La réponse instantanée d'un régénérateur à un changement en échelon est gênée par l'inertie thermique du système. On considère les régénérateurs à écoulement périodique et leur réponse transitoire à des changements en échelons simultanés de la température à l'entrée et du débit du gaz. On montre comment la réponse à un changement en échelon d'un seul des deux paramètres d'entrée peut être superposée à celle de l'autre de façon à obtenir la réponse aux deux changements simultanés.

ÜBERGANGSVERHALTEN PERIODISCHER STRÖMUNGS-REGENERATOREN

Zusammenfassung—Das sofortige Ansprechen eines thermischen Regenerators auf sprunghafte Betriebsänderungen wird durch die thermische Trägheit des Systems verhindert. Die Übergangsfunktion periodischer Strömungs-Regeneratoren bei gleichzeitigem Sprung der Gas-Eintrittstemperatur und der Gasmenge wird untersucht. Es wird gezeigt, wie die Ubergangsfunktion beim Sprung eines dieser beiden Eingangsparameter derjenigen des anderen überlagert werden kann, um die Übergangsfunktion bei gleichzeitigen Sprüngen zu erhalten.

ПЕРЕХОДНАЯ ХАРАКТЕРИСТИКА РЕГЕНЕРАТОРОВ ТЕЧЕНИЯ ПЕРИОДИЧЕСКОГО ДЕЙСТВИЯ

Аннотация — Тепловая инерция системы препятствует мгновенной реакции теплового регенератора на скачкообразное изменение параметров в процессе работы. Исследуется переходный режим работы регенераторов периодического действия при совместных скачкообразных изменениях температуры газа и его расхода на входе. Показано, как переходную характеристику при скачкообразном изменении одного из вышеуказанных параметров можно наложить на характеристику другого для получения общей характеристики совместных скачкообразных изменений в режиме работы регенератора.